# PARAMETERS OF PLANE SEPARATED FLOW

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Data of computer calculations are presented for the parameters of motion of a compressible gas in the self-similar turbulent mixing zone following separation from a solid surface. Two kinds of boundary condition are examined. On the basis of the integral relations, a calculation is made of the influence of the momentum thickness of the initial boundary layer. The results obtained may be used for calculating "closed" and "open" stagnant zones in various problems.

In a number of gasdynamic problems involving supersonic flow over steps and flat-faced bodies, and discharge of a jet into a dead space, etc., the flow is determined by turbulent mixing of the separated stream with the medium of the stagnant zone. The flow in the mixing zone may be considered self-similar except for two regions—the separated flow region in the presence of an initial boundary layer, and the attachment region of the separated flow.

For a self-similar region of flow of a compressible gas, values have been given by Neiland and Taganov [1] of the basic parameter of the stagnant zone, velocity on the dividing streamline as a function of Mach number of the external zero-gradient flow. It was shown, moreover, that there exists a spectrum of solutions of the equations of turbulent motion, covering the whole range of separated zone opening angle from the minimum to $\pi/2$.

In [2] Korst included a non-self-similar separated region in his examination. By introducing a number of assumptions, the problem of the ground pressure was represented in closed form. Solutions were found for the approximate equation of motion $u \frac{\partial u}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial y^2}$. Allowance for the influence of the initial boundary layer was made by introducing a function $f(\psi)$ into the turbulent viscosity expression $\varepsilon = \varepsilon_0 f(\psi)$. Then the function $f$ itself remains unknown and additional experimental investigations are required to determine it.

In the present paper a solution has been obtained for the complete equation of motion for plane flow of a turbulent compressible gas in the situation of the Tollmien and Dem'yanov-Shmanenkov [3] problems in the whole mixing region with $i_0$ = const and $Pr_T = 1$.

The basic equations describing plane turbulent motion of a compressible gas have the form

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial \tau}{\partial y}, \qquad (1)$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0. \qquad (2)$$

Usually, in solving (1) use is made of the Prandtl formula for the shear stress in the form

$$\tau = \pm \rho l^2 \left( \frac{\partial u}{\partial y} \right)^2,$$

where $l = x/\sigma \sim b$.

By use of the transformation

$$\xi = \int_0^y \frac{\rho}{\rho_\infty} \sigma \frac{dy}{x}$$

and by introducing the relative velocity $u/u_\infty = F'$ in the turbulent mixing zone, it is easy to obtain an ordinary differential equation from (1) and (2):

$$\frac{d}{d\xi} \left\{ \pm \left( \frac{\rho}{\rho_\infty} \right)^3 F'' |F''| \right\} - 2FF'' = 0. \qquad (3)$$

The plus sign must be taken when $F'' > 0$, and the minus when $F'' < 0$.

The relative density $\rho/\rho_\infty$, assuming constant stagnation enthalpy both in the zero-gradient region and in the mixing zone, depends only on $F'$ and the Crocco coefficient,

$$\rho/\rho_\infty = (1 - \mathrm{Cr}_\infty^2)/(1 - F'^2 \mathrm{Cr}_\infty^2).$$

The boundary conditions of the Tollmien problem are:

at the outer boundary of the zone,

$$F = \xi_1, \quad F' = 1, \quad F'' = 0;$$

at the inner boundary of the zone,

$$F' = 0, \quad F'' = 0.$$

For the Dem'yanov—Shmanenkov problem:

at the outer boundary,

$$F = \xi_1, \quad F' = 1, \quad F'' = 0;$$

at the inner boundary,

$$F = 0, \quad F'' = 0.$$

The appropriate five boundary conditions for these problems allow us to determine both the velocity profiles in the mixing zone, and the angular dimension of the zone itself:

$$\varphi_2 - \varphi_1 \left( \varphi = \sigma \frac{y}{x} \right).$$

The differential equation (3) was solved numerically on a computer. The required values of $\xi_1$ satisfying the conditions $F' = 0$ or $F = 0$ (at $\xi_2$) appropriate to the two types of boundary conditions were found by linear interpolation of the $\xi_1$ obtained from the condition that the derivative of relative velocity at $\xi_2$ equals zero.

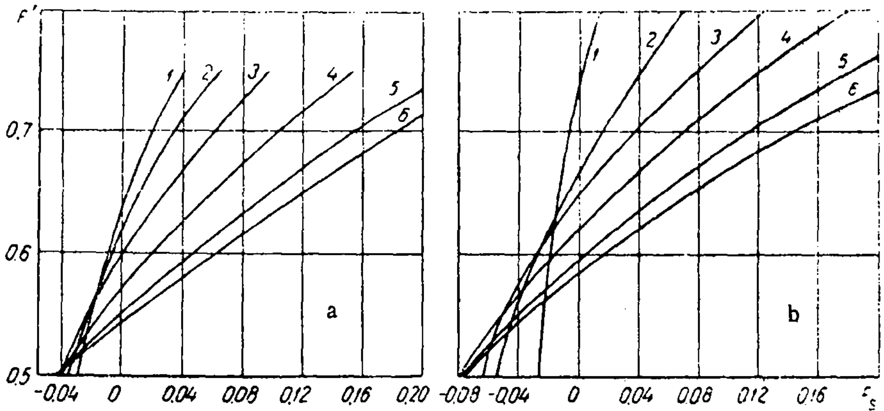


Fig. 1. Values of relative velocity on the streamlines $F_s$: a) for boundary conditions of the Tollmien problem ($1-Cr_\infty^2$ = 0.942, 2—0.843, 3—0.762, 4—0.444, 5—0.16, 6—0); b) for boundary conditions of the Dem'yanov-Shmanenkov problem ($1-Cr_\infty^2$ = 0.834, 2—0.762, 3—0.643, 4—0.444, 5—0.16, 6—0).

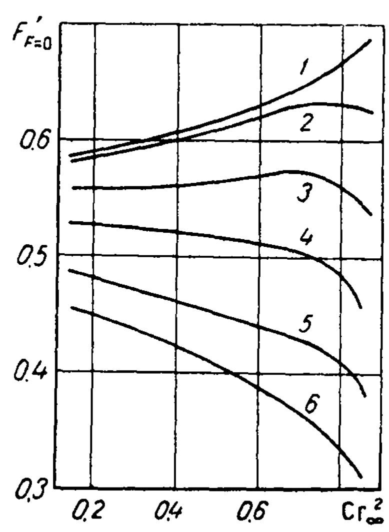


Fig. 2. Variation of $F'_{F=0}$ as a function of $Cr_\infty$: 1) with $\delta^{**}/x$ = 0; 2) 0.01; 3) 0.025; 4) 0.05; 5) 0.10; 6) 0.15.

When there is a stagnant zone, and if there is no mass flow of gas out of it, the line $F = 0$ is the dividing streamline, separating the mass of circulating gas in the stagnant zone from the mass flowing past outside the stagnant zone.

When there is a mass flow $G_s$ from the stagnant zone, this zone becomes "open" (the momentum of exiting gas is usually neglected). In this case the dividing streamline is determined from the condition

$$G_s = g u_\infty \rho_\infty \frac{x}{\sigma} F_s. \qquad (4)$$

Values of the velocity for $F = F_s$ at different Crocco numbers of the external flow are given in Fig. 1.

We shall examine the influence of the initial boundary layer on the parameters of turbulent mixing. The velocity profile in the mixing zone in the presence of an initial boundary layer becomes self-similar, beginning from a certain coordinate $x/\delta^{**}$. Then we may determine the pole of self-similarity, located at distance $x^*$ from the separation point, from the condition that the total momentum and mass flowrate of the gas at the section $x = 0$ for the true boundary layer are equal to the momentum and mass flowrate for the equivalent self-similar separated flow,

$$\left[\int_{y_2}^{y} \rho u^2 dy\right]_{eq} = \int_0^y \rho u^2 dy, \qquad (5)$$

$$\left[\int_{y_2}^{y} \rho u\, dy\right]_{eq} = \int_0^y \rho u\, dy. \qquad (6)$$

The point with this coordinate must lie outside the mixing zone or on its outer edge.

Dividing (5) by $\rho_\infty u_\infty^2$ and (6) by $\rho_\infty u_\infty$, and subtracting terms, we finally obtain

$$\delta^{**} = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) \frac{\rho u}{\rho_\infty u_\infty} dy = \left[\int_{y_2}^{y_1} \frac{\rho u}{\rho_\infty u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy\right]_{eq}.$$

Since

$$\int_{y_2}^{y_1} \frac{\rho u}{\rho_\infty u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy = \frac{x}{\sigma} \int_{F_1}^{F_2} (1 - F')\, dF,$$

the distance between the pole of the equivalent flow and the separation point may be expressed in terms of the momentum thickness of the initial boundary layer

$$x^* = \sigma\delta^{**} \Big/ \left[\int_{F_1}^{F_2} (1 - F')\, dF\right]_{eq} = \sigma\delta^{**}/I_{eq}.$$

On the basis of the mass conservation equation in the equivalent flow between the dividing streamline and the line $\psi$ passing through the lower boundary of the mixing zone, $[\psi_0 - \psi_2]_{x=0} = [\psi_0 - \psi_2]_{x \neq 0}$, we have $x^* F_2(0) = (x - x^*) F_2(x)$. Taking account of the foregoing relation, we may finally write

$$F_2 = F_2(0)/(1 + I_{eq}\, x/\sigma\delta^{**}). \qquad (7)$$

Formula (7) permits us to determine the displacement of the dividing streamline as a function of the momentum thickness of the initial boundary layer. Figure 2 gives values of the velocity on the line $F = 0$ as a function of $\delta^{**}/x$ for the boundary conditions of the Tollmien problem. In the calculations the relation $\sigma = 12 + 2.578\, M_\infty$ was used.

NOTATION

$x$, $y$—longitudinal and transverse coordinates; $u$, $v$—longitudinal and transverse velocity components; $Pr_T$—Prandtl number for turbulent boundary layer; $\delta^{**}$—momentum thickness of initial boundary layer; $b$—width of mixing zone; $\sigma$—parameter characterizing the propagation of turbulence; $i_0$—stagnation enthalpy; $Cr = \sqrt{1\Big/\left(1 + \frac{2}{\varkappa - 1}\frac{1}{M^2}\right)}$—Crocco number. Subscripts 1, 2, and $\infty$ denote, respectively, the outer and inner zone boundaries and the zero-gradient part of the flow.

REFERENCES

1. V. Ya. Neiland and G. I. Taganov, Inzh. zhurnal, 2, no. 3, 1962.

2. H. H. Korst, J. Appl. Mech, 23, no. 4, 1956.

3. Yu. A. Dem'yanov and V. N. Shmanenkov, PMM, 24, no. 2, 1960.

4. G. N. Abramovich, Theory of Turbulent Jets [in Russian], FM, 1960.

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